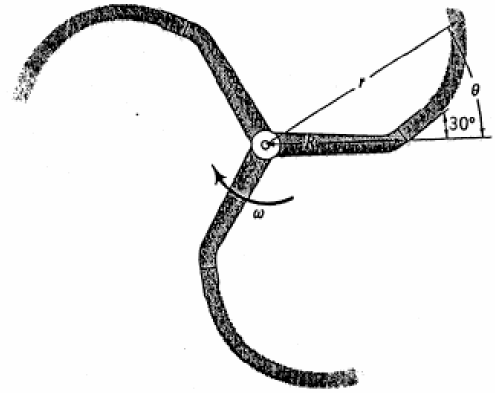
Group Homework Problems:

4.8

4.8 Water flows from a rotating lawn sprinkler as shown in **Video V4.6** and Figure P4.8. The end of the sprinkler arm moves with a speed of ωR, where $\omega = 10$ rad/s is the angular velocity of the sprinkler arm and $R = 0.5$ ft is its radius. The water exits the nozzle with a speed of $V = 10$ ft/s relative to the rotating arm. Gravity and the interaction between the air and the water are negligible. **(a)** Show that the pathlines for this flow are straight radial lines. *Hint:* Consider the direction of flow (relative to the stationary ground) as the water leaves the sprinkler arm. **(b)** Show that at any given instant the stream of water that came from the sprinkler forms an arc given by $r = R + (V_a/\omega)\theta$, where the

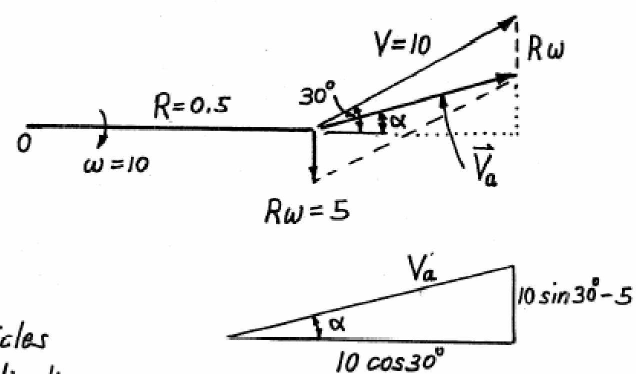


■ FIGURE P4.8

(a) Water leaves the nozzle with a velocity of $V = 10$ ft/s at an angle of $30°$ relative to the radial direction — for an observer riding on the sprinkler arm. This is the relative velocity. As shown in the sketch, the sprinkler arm has a circumferencial velocity of $R\omega = 0.5\text{ ft}\,(10\text{ rad/s}) = 5$ ft/s. The absolute velocity, $\vec{V}_a$, as observed by a person standing on the lawn is the vector sum of relative velocity and the nozzle velocity.
From the geometry of the figure:

$$\tan\alpha = \frac{10\sin 30° - 5}{10\cos 30°} = 0$$

That is $\alpha = 0$
i.e., the absolute water velocity is in the radial direction. Since there is no force acting on the water after it leaves, the water particles continue to move in the radial direction.
Thus, the pathlines are straight radial lines.

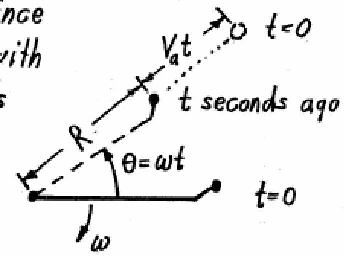


(b) The shape of the water stream at a given instant (i.e. a "snap shot" of the water) can be obtained as follows. Consider the water stream emanating from the end of the nozzle at $r = R$ and $\theta = 0$ at time $t = 0$

(con't)

4.8	(con't)

A particle in this stream that left from the nozzle t seconds ago did so when the nozzle was at $\theta = \omega t$. Since the particles in straight, radial paths with speed V_a (see part (a)), this particle is at a distance of $r = R + V_a t$ from the origin.



Thus, the stream shape is

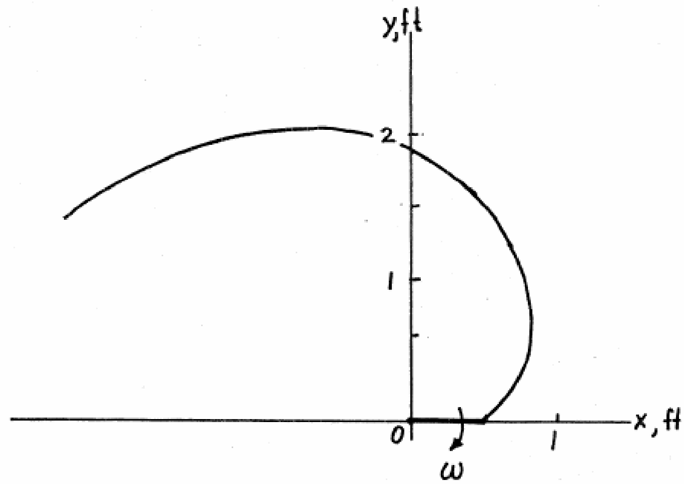
$r = R + V_a t$ and $\theta = \omega t$, or by eliminating t

$$\underline{\underline{r = R + \left(\frac{V_a}{\omega}\right)\theta}}$$

For the given data with $V_a = V\cos 30° = (10\,\frac{ft}{s})\cos 30° = 8.66\,\frac{ft}{s}$ (see part(a)) and $\omega = 10\,rad/s$ this becomes

$r = 0.5 + 0.866\,\theta$, where $r \sim ft$ and $\theta \sim rad$.

This stream shape is plotted below.



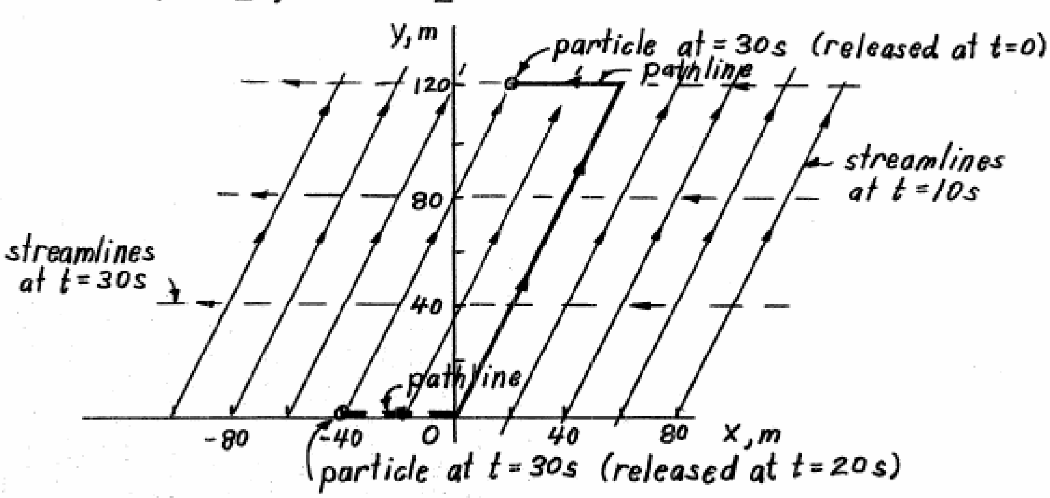
4.11 A flow in the x–y plane is given by the following velocity field: $u = 3$ and $v = 6$ m/s for $0 < t < 20$ s; $u = -4$ and $v = 0$ m/s for $20 < t < 40$ s. Dye is released at the origin ($x = y = 0$) for $t = 0$. **(a)** Draw the pathline at $t = 30$ s for two particles that were released from the origin—one released at $t = 0$ and the other released at $t = 20$ s. **(b)** On the same graph draw the streamlines at times $t = 10$ s and $t = 30$ s.

(a) For the particle released at $t=0$, $u = 3\,\frac{m}{s}$ and $v = 6\,\frac{m}{s}$ for $0<t<20s$. During this time the flow is steady and the pathline has a slope $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$. At $t=0$, $x=y=0$ and at $t=20$, $x = (3\,\frac{m}{s})(20s) = 60m$ and $y = (6\,\frac{m}{s})(20s) = 120m$

For $20<t<30$, $u = -4\,\frac{m}{s}$ and $v=0$, so that the flow is steady and the pathline has a slope of $\frac{dy}{dx} = 0$. The particle moves from $x = 60m$ to $x = 60 + (-4\,\frac{m}{s})(30-20)s = +20m$, but keeps the $y = 120m$ location during $20<t<30s$. This pathline is shown in the figure below.

For the particle released at the origin at $t = 20s$ it follows that $u = -4\,\frac{m}{s}$ and $v=0$. Thus, the corresponding pathline extends from $x=0$ to $x = (-4\,\frac{m}{s})(30-20)s = -40m$ at $t = 30s$. This pathline is shown in the figure below.

(b) At $t = 10s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$
or $y = 2x + C_1$, where $C_1 = const.$
At $t = 30s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = 0$
or $y = C_2$, where $C_2 = const.$ These lines are shown below.



4.15 Determine the acceleration field for a three-dimensional flow with velocity components $u = -x$, $v = 4x^2y^2$, and $w = x - y$.

$u = -x$, $v = 4x^2y^2$, and $w = x - y$ so that

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + (-x)(-1) + 4x^2y^2(0) + (x-y)(0) = x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x-y)(0)$$

$$= -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y-1)$$

and

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$= 0 + (-x)(1) + (4x^2y^2)(-1) + (x-y)(0)$$

$$= -x - 4x^2y^2$$

Thus,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$= \underline{\underline{x\hat{i} + 8x^2y^2(4x^2y-1)\hat{j} - (x+4x^2y^2)\hat{k}}}$$

4.23

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

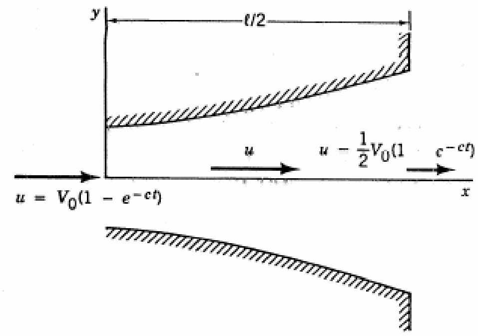


FIGURE P4.23

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$$ With $u = u(x,t)$, $v = 0$, and $w = 0$ this becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = a_x\hat{\imath}$$, where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$$a_x = V_0\left(1 - \frac{x}{\ell}\right)c\,e^{-ct} + V_0^2\left(1 - e^{-ct}\right)^2\left(1 - \frac{x}{\ell}\right)\left(-\frac{1}{\ell}\right)$$

or

$$\underline{\underline{a_x = V_0\left(1 - \frac{x}{\ell}\right)\left[c\,e^{-ct} - \frac{V_0}{\ell}\left(1 - e^{-ct}\right)^2\right]}}$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$$\left[c\,e^{-ct} - \frac{V_0}{\ell}\left(1 - e^{-ct}\right)^2\right] = 0$$ With $V_0 = 10$ and $\ell = 5$

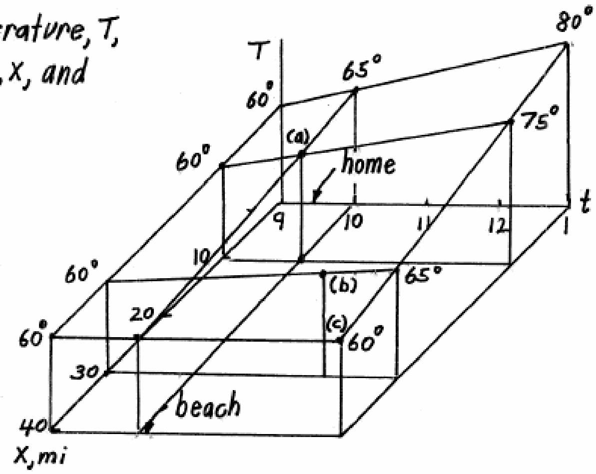
$$c\,e^{-c} - \frac{10}{5}\left(1 - e^{-c}\right)^2 = 0$$ The solution (root) of this equation is $\underline{\underline{c = 0.490\ \frac{1}{s}}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.

4.34

4.34 A bicyclist leaves from her home at 9 A.M. and rides to a beach 40 mi away. Because of a breeze off the ocean, the temperature at the beach remains 60 °F throughout the day. At the cyclist's home the temperature increases linearly with time, going from 60 °F at 9 A.M. to 80 °F by 1 P.M. The temperature is assumed to vary linearly as a function of position between the cyclist's home and the beach. Determine the rate of change of temperature observed by the cyclist for the following conditions: **(a)** as she pedals 10 mph through a town 10 mi from her home at 10 A.M.; **(b)** as she eats lunch at a rest stop 30 mi from her home at noon; **(c)** as she arrives enthusiastically at the beach at 1 P.M., pedaling 20 mph.

From the given data the temperature, T, varies as a function of location, x, and time, t, as shown in the figure.



Thus, $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$

(a) At $x = 10\,mi$ and $t = 10\,am$,

$$\frac{\partial T}{\partial t} = \frac{(75° - 60°)}{4\,hr} = \frac{15}{4}\,°/hr$$

and

$$\frac{\partial T}{\partial x} = \frac{(60° - 65°)}{40\,mi} = -\frac{1}{8}\,°/mi$$

Thus, with $u = 10\,mi/hr$,

$$\frac{DT}{Dt} = \frac{15}{4}\,°/hr + 10\frac{mi}{hr}\left(-\frac{1}{8}\,°/mi\right)$$

$$= \underline{\underline{2.5\,°/hr}}$$

(b) At noon with $u = 0$ (resting) and $\frac{\partial T}{\partial t} = \frac{(65° - 60°)}{4\,hr} = \frac{5}{4}\,°/hr$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t} = \frac{5}{4}\,°/hr = \underline{\underline{1.25\,°/hr}}$$

(c) Upon arival at the beach with $u = 20\,mph$, $\frac{\partial T}{\partial t} = 0$, and $\frac{\partial T}{\partial x} = \frac{(60° - 80°)}{40\,mi}$

$$= -0.5\,°/mi$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = u\frac{\partial T}{\partial x} = 20\frac{mi}{hr}(-0.5\,°/mi) = \underline{\underline{-10°/hr}}$$

4.46 Water flows through the slit at the bottom of a two-dimensional water trough as shown in Fig. P4.46. Throughout most of the trough the flow is approximately radial (along rays from O) with a velocity of $V = c/r$, where r is the radial coordinate and c is a constant. If the velocity is 0.04 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

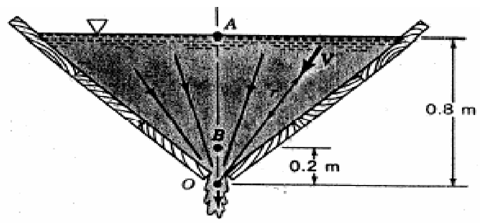


FIGURE P4.46

$\vec{a} = a_n\hat{n} + a_s\hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r}$

Since $V = 0.04\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = Vr = (0.04\frac{m}{s})(0.1m) = 4\times10^{-3}\frac{m^2}{s}$, or $V = \frac{4\times10^{-3}}{r}\frac{m}{s}$, where $r \sim m$

Thus,

$$a_s = -\left(\frac{c}{r}\right)\left(-\frac{c}{r^2}\right) = \frac{c^2}{r^3}$$

At point A:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.8m)^3} = \underline{\underline{3.13\times10^{-5}\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.2m)^3} = \underline{\underline{2.00\times10^{-3}\frac{m}{s^2}}}$$

3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

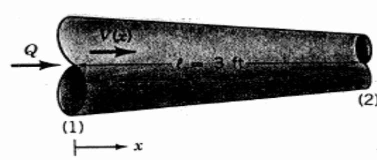


FIGURE P3.1

(a) $-\gamma\sin\theta - \frac{\partial p}{\partial s} = \rho V\frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$\frac{\partial p}{\partial s} = -\rho V\frac{\partial V}{\partial s}$ or $\frac{\partial p}{\partial x} = -\rho V\frac{\partial V}{\partial x} = -\rho(10(1+x))(10)$

Thus, $\frac{\partial p}{\partial x} = -1.94\frac{slugs}{ft^3}(10\frac{ft}{s})^2(1+x)$, with x in feet

$= \underline{\underline{-194(1+x)\frac{lb}{ft^3}}}$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1=50psi}^{p_2} dp = -194\int_{x_1=0}^{x_2=3}(1+x)dx$

or $p_2 = 50\,psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10\frac{ft}{s}$

$V_2 = 10(1+3) = 40\frac{ft}{s}$

Thus,

$$p_2 = 50\,psi + \frac{1}{2}(1.94\frac{slugs}{ft^3})(10^2 - 40^2)\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in^2}\right) = \underline{\underline{39.9\,psi}}$$

3.10

3.10 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.10. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

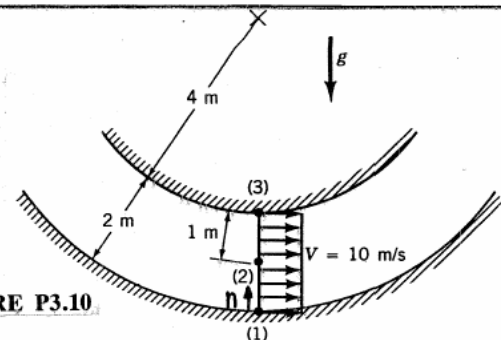


FIGURE P3.10

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{dz}{dn} = 1 \text{ and } V = 10\,m/s$$

Thus, with $\mathcal{R} = 6 - n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6-n} \quad \text{or}$$

$$\int_{n=0}^{n} \frac{dp}{dn}\, dn = -\int_{n=0}^{n} \gamma\, dn - \int_{n=0}^{n} \frac{\rho V^2 dn}{6-n}$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6-n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6-n}\right)$$

With $p_1 = 40\,kPa$ and $n_2 = 1\,m$: $p_2 = 40\,kPa - 9.8 \times 10^3 \frac{N}{m^3}(1\,m) - 999 \frac{kg}{m^3}(10\frac{m}{s})^2 \ln\left(\frac{6}{5}\right)$

or

$$\underline{\underline{p_2 = 12.0\,kPa}}$$

and

with $p_1 = 40\,kPa$ and $n_3 = 2\,m$: $p_3 = 40\,kPa - 9.80 \times 10^3 \frac{N}{m^3}(2\,m) - 999 \frac{kg}{m^3}(10\frac{m}{s})^2 \ln\left(\frac{6}{4}\right)$

or

$$\underline{\underline{p_3 = -20.1\,kPa}}$$

Individual Homework Problems:

4.7 A velocity field is given by $\mathbf{V} = x\hat{\mathbf{i}} + x(x-1)(y+1)\hat{\mathbf{j}}$, where u and v are in ft/s and x and y are in feet. Plot the streamline that passes through $x = 0$ and $y = 0$. Compare this streamline with the streakline through the origin.

$u = x$, $v = x(x-1)(y+1)$ where the streamlines are obtained from

$$\frac{dy}{dx} = \frac{v}{u} = \frac{x(x-1)(y+1)}{x} = (x-1)(y+1)$$

or $\int \frac{dy}{(y+1)} = \int (x-1)\,dx$ which when integrated gives

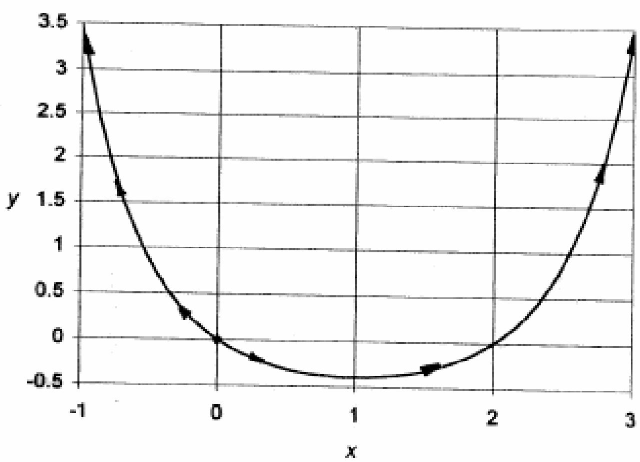
$$\ln(y+1) = \tfrac{1}{2}x^2 - x + C \text{ , where } C \text{ is a constant} \qquad (1)$$

For the streamline that passes through the origin $x=y=0$ the value of C is found from Eq. (1) as

$\ln(1) = C$, or $C = 0$

Thus, $\ln(y+1) = \frac{1}{2}x^2 - x$ or $\underline{\underline{y = e^{(\frac{1}{2}x^2 - x)} - 1}}$

This streamline is plotted below.



Note: The streamline is symmetrical about its low point of $x=1$, $y=-0.393$. At $x=y=0$ the velocity is 0.

For $x<0$, $u<0$ and for $x>0$, $u>0$. Thus, the fluid flows from the origin $(x=y=0)$.

Since the flow is steady, <u>streaklines are the same as streamlines.</u>

4.14 A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow field is the acceleration zero?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (cx^2)(2cx) = \underline{\underline{2c^2x^3}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (cy^2)(2cy) = \underline{\underline{2c^2y^3}}$$

Thus, $\vec{a} = a_x\hat{i} + a_y\hat{j} = 0$ at $\underline{\underline{(x, y) = (0,0)}}$

4.17 The velocity of air in the diverging pipe shown in Fig. P4.17 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.

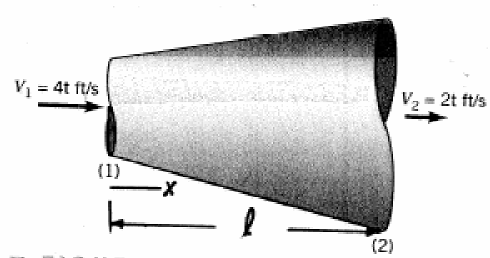


FIGURE P4.17

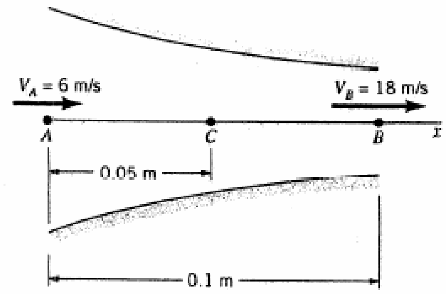
a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = \underline{\underline{4\frac{ft}{s^2}}}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = \underline{\underline{2\frac{ft}{s^2}}}$

b) convective acceleration along the pipe $= u\frac{\partial u}{\partial x}$ where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)

$$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$$

Hence, $u\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is $\underline{\underline{\text{negative}}}$.

4.21 The fluid velocity along the x axis shown in Fig. P4.21 changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.



■ FIGURE P4.21

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ With $u = u(x)$, $v = 0$, and $w = 0$

this becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = u\frac{\partial u}{\partial x}\,\hat{i} \qquad (1)$$

Since u is a linear function of x, $u = c_1 x + c_2$ where the constants c_1, c_2 are given as: $u_A = 6 = c_2$

and $u_B = 18 = 0.1 c_1 + c_2$

or $c_1 = 120$, $c_2 = 6$.

Thus, $u = (120x + 6)\frac{m}{s}$ with $x \sim m$

From Eq. (1)

$$\vec{a} = u\frac{\partial u}{\partial x}\hat{i} = (120x + 6)\frac{m}{s}\left(120\frac{m}{m \cdot s}\right)\hat{i}$$

or

for $x_A = 0$, $\vec{a}_A = 720\ \hat{i}\ \frac{m}{s^2}$

for $x_B = 0.05\,m$, $\vec{a}_B = 1440\hat{i}\frac{m}{s^2}$

and

for $x_C = 0.1\,m$, $\vec{a}_C = 2160\ \hat{i}\frac{m}{s^2}$

4.28 A company makes cars that are shipped to be sold throughout the country. At a dealership near the factory the cars cost \$20,000. At other dealerships the cost is higher because of shipping charges which are \$0.50 per mile. Determine the price of a new car at a location 800 miles from the factory and the rate of increase (\$ per hour) in the car price as it is being transported to that location on a truck traveling 55 mph on the highway. Explain your answer in terms of the material derivative.

Let x = distance from the factory, c = cost of the car, and c_0 = cost of the car at the factory. Thus, with r = rate per mile for shipping,

$c = c_0 + rx = 20{,}000 + 0.5x$, where $c \sim \$$ and $r \sim \$/mi$.

Hence, with $x = 800 mi$

$$C_{800} = 20{,}000 + 0.5(800) = \underline{\underline{\$20{,}400}}$$

and

$\frac{Dc}{Dt}$ = rate of increase of cost $= \frac{dc_0}{dt} + r\frac{dx}{dt}$

where $\frac{dc_0}{dt} = 0$ and $\frac{dx}{dt} = V = 55\ mph$

Thus,

$$\frac{Dc}{Dt} = \$0.5/mi. \left(55\frac{mi}{hr}\right) = \underline{\underline{\$27.5/hr.}}$$

In terms of the material derivative,

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = 0 + \left(55\frac{mi}{hr}\right)(\$0.5/mi) = \$27.5/hr.$$

4.36 Water flows over the crest of a dam with speed V as shown in Fig. P4.36. Determine the speed if the magnitude of the normal acceleration at point (1) is to equal the acceleration of gravity, g.

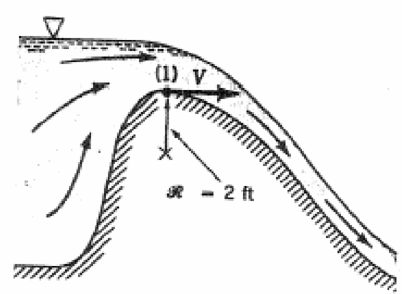
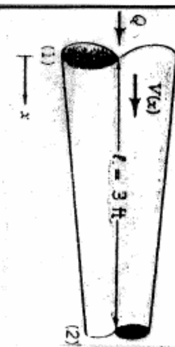


FIGURE P4.36

$$a_n = \frac{V^2}{\mathcal{R}} \text{ or with } a_n = 32.2\frac{ft}{s^2},\ V = \sqrt{a_n \mathcal{R}} = \sqrt{\left(32.2\frac{ft}{s^2}\right)(2ft)} = \underline{\underline{8.02\frac{ft}{s}}}$$

3.2

3.2 Repeat Problem 3.1 if the pipe is vertical with the flow *down*.



(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ with $\theta = -90^\circ$ and $V = 10(1+x)\,\frac{ft}{s}$

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} + \gamma$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} + \gamma = -\rho(10(1+x))(10) + \gamma$

Thus, $\frac{\partial p}{\partial x} = -1.94\,\frac{slugs}{ft^3}\left(10\,\frac{ft}{s}\right)^2(1+x) + 62.4\,\frac{lb}{ft^3}$, with x in feet

$$= \underline{\underline{-194(1+x) + 62.4\,\frac{lb}{ft^3}}}$$

(b)(i) $\frac{dp}{dx} = -194(1+x) + 62.4$ so that $\int_{p_1=50psi}^{p_2} dp = \int_{x_1=0}^{x_2=3} [-194(1+x) + 62.4]\,dx$

or $p_2 = 50psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) + 62.4(3)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right)$

$= 50 - 10.1 + 1.3 = \underline{41.2\,psi}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = 0$, $z_2 = -3\,ft$

and $V_1 = 10(1+0) = 10\,\frac{ft}{s}$, $V_2 = 10(1+3) = 40\,\frac{ft}{s}$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) - \gamma z_2$

$= 50psi + \frac{1}{2}\left(1.94\,\frac{slug}{ft^3}\right)(10^2 - 40^2) - 62.4\,\frac{lb}{ft^3}(-3\,ft)$

$= \underline{\underline{41.2\,psi}}$

3.5 At a given location the air speed is 20 m/s and the pressure gradient along the streamline is 100 N/m³. Estimate the air speed at a point 0.5 m further along the streamline.

If neglect gravity, $\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial V}{\partial s} = -\frac{\partial p}{\partial s}/\rho V$

or $\frac{\partial V}{\partial s} = -100\,\frac{N}{m^3}\Big/\left(1.23\,\frac{kg}{m^3}\right)\left(20\,\frac{m}{s}\right) = -4.07\,\frac{1}{s}$

Thus,

$\delta V \approx \frac{\partial V}{\partial s}\,\delta s = \left(-4.07\,\frac{1}{s}\right)(0.5m) = -2.03\,\frac{m}{s}$, so that $V + \delta V = 20\,\frac{m}{s} - 2.03\,\frac{m}{s}$

or $\underline{\underline{V \approx 18.0\,\frac{m}{s}}}$